DOCUMENT RESUME

ED 335 215 SE 052 174

AUTHOR Kouba, Vicky L.; McDonald, Janet L.
TITLE A Model for Inferring Components of Belief Systems by Interpreting the Content and Context Cues Used by Students to Situate Mathematical Tasks.
PUB DATE Apr 91
NOTE 24p.; Paper presented at the Annual Meeting of the American Educational Research Association (Chicago, IL, April 3-7, 1991).
PUB TYPE Information Analyses (070) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Beliefs; *Cognitive Development; *Context Effect; *Cues; Elementary School Mathematics; Elementary Secondary Education; Learning Processes; Mathematical Linguistics; Mathematics Education; Mathematics Instruction; Mathematics Teachers; *Performance Factors; Secondary School Mathematics; Teaching Methods
IDENTIFIERS *Constructivist Theory; Mathematics Education Research

ABSTRACT

Confrontation with a mathematical task causes a student to focus initial attention upon particular content and context cues based upon that student's belief system; the student assigns subsequent meaning to that task based upon previous mathematical experience and knowledge. In order to make effective use of students' belief systems, educators need delineations both of the components of those systems and of the interactions of those systems with students' cognition. A model is presented for the inference of the positive and negative components of students' belief systems by interpretation of the content and the context cues that students use within their mathematical identification and construction techniques. Three assumptions that parallel those concerning teachers' belief systems are noted: (1) notions and beliefs are implicit within responses to questions or issues related to mathematical tasks and hence can be identified and described; (2) some beliefs can impede effective and efficient mathematical understanding and performance; and (3) eventually, educators may be able to modify instruction based upon knowledge of the components of students' belief systems. (33 references) (JJK)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED335215

A Model for Inferring Components of Belief Systems By Interpreting the Content and Context Cues Used by Students to Situate Mathematical Tasks

Vicky L. Kouba
The University at Albany
State University of New York

and

Janet L. McDonald
Union College
Schenectady, New York

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

☐ This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
Vicky L. Kouba
Janet L. McDonald
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

A paper presented at the annual meeting of the American Educational Research Association, Chicago, Illinois, April, 1991

SE 052 174

BEST COPY AVAILABLE

A Model for Inferring Components of Belief Systems By Interpreting the Content and Context Cues Used by Students to Situate Mathematical Tasks

When students come to a mathematical task their first action is to assign meaning to the task based on their experience and knowledge of the content and context of that task. Part of the process of assigning meaning is to determine whether or not the task is mathematical. When students situate tasks[1] in the mathematical domain they cue on the content and context features of the task. Their belief systems determine what content and context cues they attend to. Thus, these belief systems affect how students solve problems and how students think about mathematics (Schoenfeld, 1988; Underhill, 1988).

But it is not enough simply to say that belief systems affect what students attend to (Ginsburg & Asmussen, 1988; Schoenfeld, 1985). To make effective use of those systems, educators need delineations of the components of the systems, as Thompson (1988) has provided for teachers' belief systems. Our assumptions about the interaction of beliefs and cognition parallel Thompson's: that notions and beliefs are implicit in responses to questions or issues related to mathematical tasks and thus can be identified and described; that some beliefs

---

[1] Here we intend the meaning of situated knowledge to be similar to how Lave (1988) and Brown, Collins, & Duguid (1989) use the term, except that we are considering the way that knowledge is situated just in the domain of mathematics rather than the whole sphere of situated knowledge.

impede effective and efficient mathematical understanding and performance; and that eventually we may be able to modify instruction based on knowledge of the components of the belief systems.

In this paper, we present a model for inferring the positive and negative components of students' belief systems by interpreting the content and context cues that they use to determine when to reach for their mathematical toolbox. We anticipate that work in identifying the components of students' belief systems will be the basis for successful modification of instruction just as knowledge of how students learn has been used successfully to modify instruction. For example, the Cognitively Guided Instruction Project reported positive results when teachers were trained and encouraged to use their knowledge of students' learning of addition and subtraction to make instructional decisions (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). An analysis of the components of students' belief systems, therefore, should enable educators to help students reconstruct their belief systems by building on the positive components and building around the negative components.

The Model

The overall scheme of the model for inferring components of students' belief systems by interpreting cues of situated mathematical tasks (ICSMT) is shown in Figure 1. The top portion of the model is an extension and adaption of the student portion of a model for research and curriculum development presented by

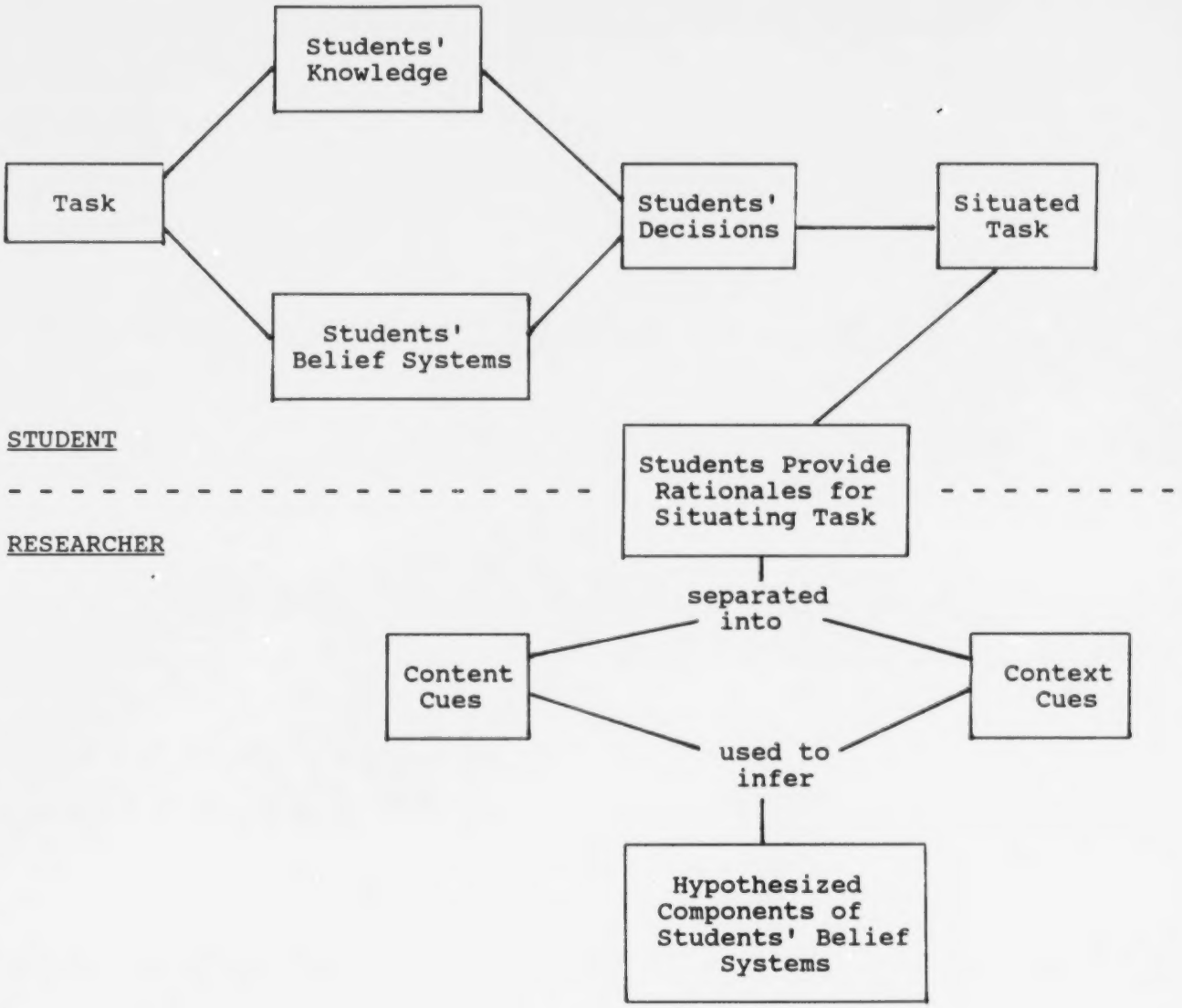

Figure 1. A model for inferring components of students' belief systems by interpreting cues of situated mathematical tasks (ICSMT).

Carpenter and Fennema (1988). The top portion of the model depicts the students' processing of the task showing that their decisions about situating that task are affected by their mathematical knowledge and belief systems. The bottom portion of the ICSMT model depicts the process that may be applied to interpret the students' rationales for situating tasks. The students' rationales for classifying a task as mathematical are analyzed, clustered, and separated into content cues and context cues. These cues are then analyzed and clustered, and the cluster used to infer components of students' belief systems.

The term "content" is used here much as Kulm (1984) uses it as a label for a category of task variables. So by "content" we refer to mathematical meanings of the situation. This would include, for example, the content area or strand (measurement, probability, and geometry), type of operation involved, and the complexity of the problem (single step, multi-step). Unlike Kulm, we include syntax (symbols, terms, and the size or type of the number) as part of content.

By "context" we refer to nonmathematical or incidental meanings in the situation as described by Webb (1984) where:

> ...the context refers to the <u>form</u> of the problem statement. "Form" is interpreted very generally to include: (1) variables describing the problem embodiment or representation, (2) variables describing the verbal context or setting, and (3) variables describing the information format. (p. 69)

We extend our use of context to include aspects that are not observable in a task, for example, where and when a concept or process was learned.

Identifying Components of Student Belief Systems

In our application of the ICSMT model, we asked students to provide rationales for situating a task in the mathematical domain. In a series of studies (Kouba & McDonald, 1987; Kouba & McDonald, in press; McDonald & Kouba, 1986) we gave 2,703 K-8 students a set of situations and asked them to identify whether each involved mathematics and to explain their answers. Sample situations are shown in Figure 2. We identified patterns in the most frequent response types. These patterns were verified by classroom teachers, mathematics educators, and mathematicians. We examined the responses within each pattern to identify what the students were cueing on and categorized those as content or context cues (see Figures 3 and 4).

The components of the students' belief systems were inferred from straightforward statements of beliefs and implied statements of beliefs expressed in the rationales grouped under each content and context cue. The straightforward statements of belief included such expressions as, "It's math because there are numbers and numbers are math." The implied statements of belief included such expressions as, "It's not math because it doesn't have a problem part," which we interpreted as a belief that mathematics consists of problems of a particular form and presentation. The components then were classified as being

1. Alan took out his ruler and measured his desk.

2. Leslie said when she rolled the two dice that the numbers would more likely add to seven than to twelve.

3. Raphael studied the two cornfields to see if the distance around their borders could be the same if the shapes were different.

4. In gym class Ryan drew diagrams to show the changes in his heartrate before and after exercising.

5. Natalie helped her mother decide how much carpet they should buy for the livingroom.

6. Ellen read a book about trees.

7. Melanie had to tell the teacher which was greater, 5 or 3.

8. Fran told the teacher which figures were squares in the picture on the page and which were not.

Figure 2. Sample situations used to elicit rationales for situating mathematical tasks.

CONTENT CUES	POSITIVE COMPONENT	NEGATIVE COMPONENT
Presence or absence of numbers	Numbers play an important role in mathematics.	Numbers are isomorphic with mathematics.
Presence or absence of operations	Operations play an important role in mathematics.	Operations are isomorphic with mathematics.
Presence of charts, diagrams, or graphs	Charts, graphs, and diagrams are mathematical tools.	Charts, graphs, and diagrams are not mathematical tools.
Inclusion of a particular strand	Mathematics includes a particular strand.	Mathematics is comprised of a limited number of strands.
Presence of mathematical terms	Mathematics is a language.	Key words are the most important part of problem solution.
Nature of the process and the product	Processes in mathematics are as important as products. Mathematics consists of more than finding right answers.	Mathematics must be an active process which results in only one right answer.
Perceived level of complexity or difficulty	Mathematics consists of concepts that range from simple to complex and easy to hard.	Mathematics consists of only complex and difficult concepts and processes and cannot be automatic.

Figure 3. Content cues and inferred positive and negative belief components.

CONTEXT CUES	POSITIVE COMPONENTS	NEGATIVE COMPONENTS
Form and presentation of the problem	Form and presentation give clues to algorithms. Mathematics comes in many forms and presentations.	Mathematics consists solely of "type" problems. The form and presentation are sufficient to determine the algorithm or approach.
Where the mathematics was learned or used	Mathematics can be learned and used in many places.	Mathematics is what is learned and used in mathematics class. Content areas are isolated domains.
Grade level when the mathematics was learned	Mathematics is a cumulative body of knowledge.	If something was learned in a previous grade and is no longer practiced, then it is no longer mathematics.
Presence of manipulatives or tools	Certain manipulative tools are mathematical.	Use of certain manipulatives or tools precludes mathema ical activity.

Figure 4. Context cues and inferred positive and negative belief components.

either positive or negative and expressed as statements that are building blocks for belief systems (see Figures 3 and 4). By positive we imply that the belief is one which would commonly be held by mathematicians and mathematics educators. By negative we refer to misconceptions or incomplete conceptions of mathematics, which might impede effective or efficient mathematical understanding or performance.

Content Cues. We first identified pairings of positive and negative components of belief systems for each set of content cues used by the students (see Figure 3). What follows is a discussion of the implications of those belief components.

The presence of numbers is a cue used frequently by the students in our studies to determine whether or not a situation is mathematical. Although it is important to recognize that numbers play a significant role in mathematics, identifying numbers as a necessary and sufficient condition for recognizing a situation as mathematical causes some students to see numbers as the only products of mathematics. Students who focus on numbers fail to see other mathematics when numbers are not present or fail to go beyond the number when numbers are just superficial components. As a result, these students fail to identify geometry, probability, or complex, multi-level tasks as mathematical.

This narrow view of mathematics may have a debilitating effect on students' mathematical performance. In Cobb's (1985) study of two children's problem solving, one child, Scenetra,

appeared to be so bound by her focus on numbers and numerals themselves that she was unable to solve problems that relied on the analysis of relationships and patterns of relationships among numbers.

The presence or absence of arithmetic operations is a second primary content cue used by students to situate a task in the mathematical domain. Focusing on operations (also noted by Frank, 1988 and Joffe & Foxman, 1984) may further limit the students' ability to recognize the presence of non-arithmetic mathematics. It also may limit their repertoire of problem-solving strategies. Rather than try to make sense of a problem, these students may try to reduce problem solving to just the identification of an operation. The prevalence of such immature strategies has been described by Sowder (1988).

A third content cue used by students is the presence of charts, diagrams and other forms of data presentation. Students who persist in recognizing mathematics in situations where charts, diagrams, and graphs are present but numbers are not explicit apparently include these representations of data as tools in the mathematical domain. Students who do not recognize charts, diagrams, and graphs as tools belonging to the mathematical domain may believe that representations of data are merely descriptive. As mere descriptions, these representations may not be viewed by the students as providing data to which mathematical operations can be applied. This rejection of the data as something that can be operated on mathematically may

explain students' poor performance on data analysis portions of national assessments of mathematics (Brown, Carpenter, Kouba, Lindquist, Silver, & Swafford, 1988; Kouba, Brown, Carpenter, Lindquist, Silver & Swafford, 1988) and therefore warrants further research.

Students who include mathematical strands (a fourth content cue) which go beyond numbers and operations have a broader and possibly more integrated view of mathematics as has been recommended by the Standards (NCTM, 1989). For students who exclude strands other than numbers and operations, mathematics becomes synonymous with arithmetic. These students can look at a situation, see and even name strands other than numbers and operations (e.g. probability, geometry, or measurement), yet they reject such strands as being mathematical. As a consequence, they are as limited as those students who don't look beyond the numbers.

A fifth content cue is the presence of mathematical terms. Students who identify a situation as mathematical because of the terms that are used are acknowledging that those terms have mathematical definitions. Such an acknowledgement is a prerequisite to understanding that mathematics is a language. Treating mathematics as a language enables students to develop the linguistic networks and registers (Halliday, 1978) which are helpful in analyzing problems, making mathematical sense of situations, and making choices of algorithms or procedures. Acknowledging that certain terms have precise mathematical

meanings is, however, different from focusing on those terms in isolation. Isolating terms may lead students to respond inappropriately to mathematical situations. This rote reliance on "key word" analysis has been described and criticized at length (Nesher & Teubal, 1975; Schoenfeld, 1982; Sowder 1988). Recently Stockdale (1991) analyzed a sample of current textbooks and found that the language of word problems has been changed so that the use of a rote "key word" strategy will no longer yield a high performance score. This may help in preventing the coming generation of students from developing a belief that rote use of "key words" is a "good" strategy.

A sixth content cue category is the nature of the process and product in mathematics. Believing that mathematics is more than just finding answers enables students to expand their domain of mathematics to include such processes as estimating, comparing, and finding patterns. These processes are efficient tools for making connections from one problem situation to another and from one mathematical context to another. In Cobb's (1985) study of two children, the child who was able to focus on processes and to view mathematics as more than just finding a numerical answer was more able to solve novel problems. Borasi (1990) has identified four dysfunctional mathematical beliefs that she links to product and process and explains how these beliefs not only result in misconceptions but also lead to a dualistic view of mathematics.

Students for whom the process of mathematics must result not

just in finding answers but in finding one right answer appear to fixate on the final product. Frank (1988) and Borasi (1990) describe how this focus can translate to a dichotomized view of mathematics as being either completely right or completely wrong. Here the process is not only ignored, but is also viewed as valueless if the product is determined to be wrong. Ginsburg and Asmussen (1988) identify students who believe the process to be "thoughtless," with the right answer derived either through computation or memory.

Other students attend to the process, but hold the view that it must be an active one; the action involved being limited to physically performing calculations. Active does not mean intellectually engaged or using manipulatives. It means using paper and pencil to write out calculations. Ginsburg and Asmussen (1988) describe children who view reasoning about a problem as cheating and who would perform tedious computations even if they could eliminate them by applying reason or understanding of the mathematical principle.

A final content cue used by students to situate mathematical tasks is the perceived level of complexity or difficulty of the task. When the processing related to the situation is considered to be automatic or the content of the situation is considered to be easy or already known, some students refuse to consider the situation mathematical. This view of mathematics as an "upwardly shifting domain" may have a direct effect on students' confidence in their ability to do mathematics because mathematics becomes,

by definition, only that which is hard or unknown. While we know that it is important that students automatize we must be aware that for some students reaching a level of automization or the development of number sense may result in the devaluing of those abilities. Likewise, common sense and considering the reasonableness of solutions may not be perceived as mathematical.

It is possible that when we deny students their "fall back" strategies of counting on their fingers or the early strategies in Siegler and Shrager's (1984) model of strategy choice, we may be fostering the belief that those strategies are not mathematics. This, in turn, may limit the student's repertoire of strategies and thus, their flexibility in solving problems. Ginsburg and Asmussen (1988) describe their analysis of a thirty-year old woman's (Jessica) mathematical processing. In studying Jessica they found that her performance was adversely affected by the belief that mathematics must always be done with hard, complex algorithms. The simple arithmetic and measuring that she could do well, she did not consider to be mathematics. She therefore neither gave herself credit for the mathematical skills that she had nor did she use them.

Context Cues. We next identified pairings of positive and negative components of belief systems for each set of context cues which were used by the students in our studies (see Figure 4). Below we discuss the implications of those belief components.

Students often use the form or presentation of a problem to

situate a mathematical task. Students who can categorize problems and match them to effective solution strategies are more likely to be able to solve problems. However some students become too rigid in setting their parameters for what constitutes a mathematical problem and fail to recognize mathematics in anything but "textbook-like" situations. Thus, they lose the opportunity to use mathematics to make sense of situations (Kaput, 1989) that cannot be readily "typed."

For some students the form is all that is important. As long as there are two or more numbers and a question that calls for a numerical answer the students perceive the problem as mathematical and will process the numbers even if the problem makes no sense (Schoenfeld, 1988).

A second contextual cue used by students is the location where mathematics is learned or applied. This includes the actual physical location where the mathematics is learned, such as the mathematics classroom, as well as the contextual location in which mathematics problems are placed, i.e. "real world" problems versus non-applied problems, or mathematics problems versus science or social studies problems.

Students who focus entirely on the physical environment in which the learning has taken place are unlikely to be able to transfer their learning to other environments. It is possible that dividing the elementary school day into "math time" and other "times" and dividing the school day into subject "periods" at higher levels may contribute to this rigid partitioning which

students adhere to. Also, as suggested in the Standards (NCTM, 1989), not enough time is spent helping students make the link between simplified "purely mathematical" situations and their applied counterparts.

Several researchers (Carraher, Carraher & Schliemann, 1985, 1987; Lave, 1988; Saxe, 1988) describe students who distinguish either implicitly or explicitly between "school math" and "street math." The same students who are capable of selecting and applying appropriate algorithms (often student-derived algorithms) and obtaining correct solutions in arenas outside of the mathematics classroom are less successful when given comparable or even the same problems in a mathematics classroom or school setting. The reverse is also true. Many students do not make the connection between the mathematical concepts and skills they learn in the classroom and situations in daily life where those skills might be helpful. While some student believe there are two separate mathematical arenas (school and daily life), others believe there is only one, that of the mathematics classroom which consists of teacher-directed paper and pencil activities.

A third context-related cue students use to situate mathematics is the grade level when the mathematics was learned. The belief that mathematics is a cumulative body of knowledge may provide an intrinsic motivation for students to seek connections within the mathematics they are learning and to construct logical frameworks of knowledge about mathematics. On the other hand,

students who believe mathematics is bounded, not only by where it is learned, but by when it is learned, may have fewer reasons or internally controlled goals for structuring mathematics. These students view mathematics as temporal. That is, what was mathematics last year may not be mathematics this year unless it is something practiced within this year's mathematics class. A time-bound view of mathematics may be encouraged by a fragmented organization of the curriculum and by changes in the "rules" of mathematics, such as when justifying regrouping in subtraction we tell younger students that they cannot subtract a smaller number from a larger number and then do just that in later grades when working with signed numbers.

Students holding the belief that mathematics is only what is learned or practiced at a specific grade level may have little personal ownership of mathematical knowledge. Not only is mathematics what the teacher controls, but it is what the current teacher controls. Certain teaching practices, such as insisting that students solve a specified way this year regardless of the strategies they may have learned previously, may foster this time-bound view of mathematics. Borasi (1990) found many students who have such a cold, impersonal, non-ownership image of mathematics. Further research is needed to determine if the manifestation of these time-bound, impersonal beliefs can be avoided when mathematics instruction, such as that advocated by the NCTM Standards (1989) and that used in Cognitively Guided Instruction (Fennema & Carpenter, 1989), which encourage students

to solve problems in their own ways and in a variety of ways.

A final context cue used by students to decide if a task is mathematical is the presence of manipulatives or tools. Students who accept the use of manipulatives or tools increase the number of and the likelihood of using their fallback strategies and technological strategies to solve routine and novel problems. The exclusion of manipulatives and tools from mathematics is a debilitating belief that may be the result of believing that the action of performing the calculation <u>is</u> the mathematics. If a calculator is used then no mathematics is performed even though the student sets up the calculation, enters the numbers and is left to make sense of and apply the answer. These latter processes are not viewed as mathematical. The answer itself, although a number, loses its power as a cue. The context of the calculator overshadows the content of the numbers because the use of the tools is viewed as a form of cheating. To make use of the calculator (or other tool) circumvents the mathematics.

Summary

We concur with Underhill (1988), "All knowledge is a set of beliefs." (p.63) Thus, beliefs about mathematics can have a powerful influence on how students learn and use mathematics. By systematically identifying, classifying and analyzing the content- and context-based choices students make about what mathematics is, we are able to infer components of the students' belief system. In turn, knowledge about these components may help us to identify possible causes for debilitating beliefs and

may suggest ways to enhance positive beliefs and alter or work around negative beliefs.

References

Borasi, R. (1990). The invisible hand operating in mathematics instruction: Student's conceptions and expectations. In T. J. Cooney & C. R. Hirsch (Eds.), Teaching and Learning Mathematics in the 1990s: 1990 Yearbook of the National Council of Teachers of Mathematics (pp. 174-82), Reston, VA: The Council.

Brown, C. A., Carpenter, T. P., Kouba, V. L., Lindquist, M. M., Silver, E. A. & Swafford, J. O. (1988a). Secondary school results for the fourth NAEP mathematics assessment: Discrete mathematics, data organization and interpretation, measurement, number and operations. Mathematics Teacher, 81 (4), 241-248.

Brown, J. S., Collins, A. & Duguid, P. (1989). Situated cognition and the culture of learning. Educational Researcher. 18 (1), pp. 32-42.

Carpenter, T. P. and Fennema, E. (1988). Research and cognitively guided instruction. In E. Fennema, T.P. Carpenter, and S. J. Lamon (Eds.) Integrating research on teaching and learning mathematics. Madison, WI: National Center for Research in Mathematical Sciences Education, Wisconsin Center for Education Research.

Carpenter, T.P., Fennema, E., Peterson, P.L., Chiang, C., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. American Educational Research Journal, 26, 385-401.

Carraher, T. N., Carraher, D. W. & Schliemann, A. D. (1985). Mathematics in the streets and in the schools. British Journal of Developmental Psychology, 3, 21-29.

Carraher, T. N., Carraher, D. W. & Schliemann, A. D. (1987). Written and oral mathematics. Journal for Research in Mathematics Education, 18 (2), 83-97.

Cobb, P. (1985). Two children's anticipations, beliefs, and motivations. Educational Studies in Mathematics, 16 (2), 111-126.

Fennema, E. & Carpenter, T. (1989). Cognitively Guided Instruction: A program implementation guide. Madison, WI: Wisconsin Center for Education Research.

Frank, M. L. (1988). Problem solving and mathematical beliefs. Arithmetic Teacher, 35 (5), 32-34.

Ginsberg, H. P. & Asmussen, K. A. (1988). Hot mathematics. In G.B. Saxe and M. Gearhart (Eds.) Children's Mathematics, (pp. 89-111). San Francisco, CA: Jossey-Bass Inc.

Halliday, M. A. K. (1978). Language as social semiotic. London: Edward Arnold.

Joffe, L. & Foxman, D. (1984). Assessing mathematics 5. Attitudes and sex differences: Some APU findings. Mathematics in School, 13 (4), 22-25.

Kaput J. J. (1989). Information Technologies and affect in mathematical experience. In D. B. McLeod and V. M. Adams (Eds.), Affect and mathematical problem: A new perspective, (pp. 89-103). NY: Springer-Verlag.

Kouba, V. L., Brown, C. A., Carpenter, T. P., Lindquist, M. M., Silver, E. A. & Swafford, J. O. (1988). Results of the fourth NAEP assessment of mathematics: Measurement, geometry, data organization, attitudes, and other topics. Arithmetic Teacher, 35 (9), 10-16.

Kouba, V. L. & McDonald, J. L. (1987). Students' perceptions of mathematics as a domain. In J. C. Bergeron, N. Herscovics, & C. Kieran (Eds.) Psychology of mathematics education, PME-XI, Vol. 1 (pp. 106-112), Montreal, Canada:PME.

Kouba, V. L. & McDonald, J. L. (in press). What is mathematics to children? The Journal of Mathematical Behavior, 10 (2).

Kulm, G. (1984). The classification of problem-solving research variables. In G. A. Goldin and C. E. McClintock (Eds.) Task Variables in Mathematical Problem Solving (pp. ~~69-102~~ 1-22). Philadelphia, PA: The Franklin Institute Press.

Lave, J. (1988). Cognition in Practice: Mind, Mathematics and Culture in Everyday Life. Boston, MA: Cambridge University Press.

McDonald, J. L. & Kouba, V. L. (1986). Children's and teachers' perceptions and beliefs about the domain of elementary mathematics. In G. Lappan (Ed.) Proceedings of the eighth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (pp. 250-255). East Lansing, MI.

McLeod, D.B. (1989). Beliefs, attitudes, and emotions: New views of affect in mathematics education. In D. B. McCloud and V. M. Adams (Eds.) Affect and Mathematical Problem Solving: A New Perspective (pp. 245-258). NY: Springer-Verlag.

National Council of Teachers of Mathematics (1989). Curriculum and Evaluation Standards for School Mathematics. Reston, VA: The Council.

Nesher, P. & Teubal, E. (1975). Verbal cues as an interfering factor in verbal problem solving. Educational Studies in Mathematics, 6, 41-51.

Saxe, G. B. (1988). Candy selling and math learning. Educational Researcher, 17 (6), 14-21.

Schoenfeld A. H. (1982). Some thoughts on problem-solving research and mathematics education. In F. K. Lester and J. Garofalo (Eds.), Mathematical problem solving: Issues in research. (pp. 27-37). Philadelphia, PA: The Franklin Press.

Schoenfeld, A. H. (1985). Mathematical problem solving. San Diego: Academic Press.

Schoenfeld, A. H. (1988). Problem solving in context(s). In R. I. Charles and E. Silver (Eds.) The Teaching and Assessing of Mathematical Problem Solving (Volume 3) (pp. 82-92). Reston, VA: The National Council of Teachers of Mathematics and Lawrence Erlbaum Associates.

Siegler, R. S., & Shrager, J. (1984). A model of strategy choice. In C. Sophian (Ed.), Origins of cognitive skills. (pp. 229-293). Hillsdale, NJ: Erlbaum.

Sowder, L. (1988). Choosing operations in solving routine story problems. In R. I. Charles and E. A. Silver (Eds.) The Teaching and Assessing of Mathematical Problem Solving (Volume 3) (pp. 148-158). Reston, VA: The National Council of Teachers of Mathematics and Lawrence Erlbaum Associates.

Stockdale, S. R. (1991). A study of frequency of selected cue words in elementary textbook word problems. School Science and Mathematics, 91 (1), 15-21.

Thompson, A. G. (1988). Learning to teach mathematical problem solving: Changes in teachers' conceptions and beliefs. In R. I. Charles and E. Silver (Eds.) The Teaching and Assessing of Mathematical Problem Solving (Volume 3) (pp. 82-92). Reston, VA: The National Council of Teachers of Mathematics and Lawrence Erlbaum Associates.

Underhill, R. (1988). Focus on research into practice in diagnostic and prescriptive mathematics: Mathematics learners' beliefs: A review. Focus on Learning Problems in Mathematics, 10(1), 55-69.

Webb, N. (1984). Content and context variables in problem tasks. In G. A. Goldin and C. E. McClintock (Eds.) Task Variables in Mathematical Problem Solving (pp. 69-102). Philadelphia, PA:The Franklin Institute Press.